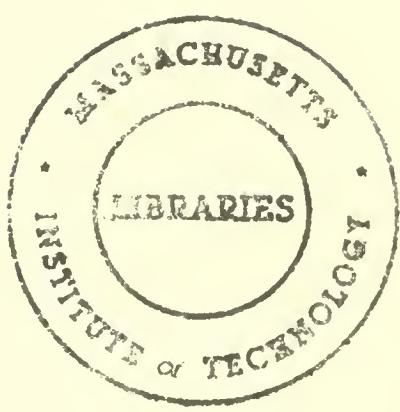


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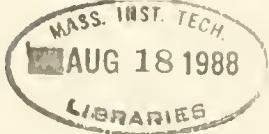






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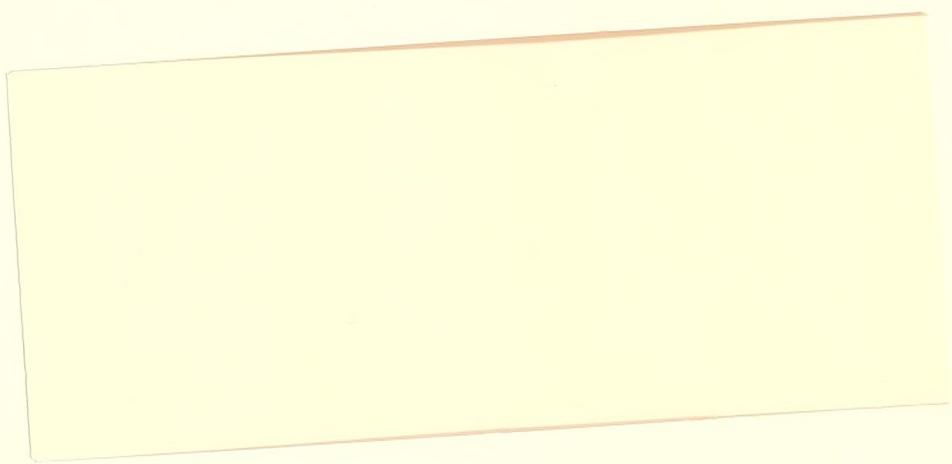
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COLLUSIVE PRICE LEADERSHIP

By  
Julio J. Rotemberg  
and  
Garth Saloner\*

Working Paper #2003-88  
Revised March 1988

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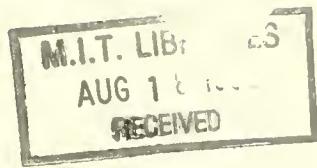
COLLUSIVE PRICE LEADERSHIP

By

Julio J. Rotemberg  
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\*M.I.T. We would like to thank the participants of the GTE Labs. Economics symposium (August 1985) for useful comments and the National Science Foundation (grants SES-8209266 and IST-8510162 respectively) for financial support.



## I. INTRODUCTION

In many industries pricing is characterized by price leadership: one of the firms announces a price change in advance of the date at which the new price will take effect and the new price is swiftly matched by the other firms in the industry. Strikingly, a long time elapses between price changes (often a year or more) and price changes are usually matched to the penny even when the products are differentiated.

Examples of this pattern of pricing behavior abound. Perhaps the best known example occurred in the cigarette industry in the late 1920's and early 1930's. For instance, on October 4, 1929 Reynolds announced an increase in its price from \$6.00 to \$6.40 per thousand (effective the October 5) and was followed the next day by both of its major competitors, Liggett and Meyers and American Tobacco. That price was in effect for almost two years before Reynolds led a further increase (to \$6.85).<sup>1</sup> Similar pricing behavior has been documented in the steel, dynamite, anthracite and airline industries.<sup>2</sup>

Markham (1951) suggests that this pattern of pricing is "price leadership in lieu of overt collusion". That is, since meetings in "smoke-filled rooms" to reach price agreements violate the antitrust laws, firms use these public announcements to achieve collusive pricing coordination. Despite the ubiquity of the pricing pattern and the importance of the collusive interpretation, there has been no attempt to examine the economic properties of collusive price leadership. This paper is devoted to a theoretical investigation of those properties and an empirical study of a recent example in the airline industry.

We examine a differentiated products duopoly in which the firms are asymmetrically informed. We investigate a price leadership scheme in which pricing decisions are delegated to the better informed

firm (the "leader") who announces pricing decisions ahead of time. For its part, the follower is expected to match these prices exactly.

This scheme has a number of positive attributes from the point of view of the duopoly: it is extremely easy to implement; defining adherence to the scheme is trivial in the sense that there is no ambiguity as to the desired response of the follower; no overt collusion (either through information transfer or price-fixing) is required and, while the scheme is generally not optimal, both firms enjoy responsiveness to demand conditions since prices embody the leader's superior information.

We begin by providing the conditions under which such price leadership can be sustained as a collusive equilibrium in a repeated game by the (credible) threat that the industry will revert to uncooperative behavior if any firm "cheats" on the collusive understanding. Since the leader's price is followed in equilibrium, it obviously chooses the industry price it most desires.

The substantive equilibrium condition concerns whether the follower has an incentive to deviate. As is usual for games of this kind, the critical condition trades off the one-period gain from unilaterally deviating against the future costs of the breakdown in cooperation. However, the calculation of the incentive to deviate is complicated by the fact that the price announcement may reveal some of the leader's private information. As a result the follower's calculation involves a signal extraction problem.

Our main focus is on the characteristics of the equilibrium rather than the existence conditions, however. The main features of the equilibrium are the following:

First, if the firms produce differentiated products and a common price is charged, they will typically have differing preferences about what that price should be. Since the firm that is

the designated price leader typically earns higher profits under these circumstances, it might be expected that both firms would vie for the leadership position. We show, however, that when one of the firms is better informed than the other, the latter may prefer to follow than to lead. Thus the industry leader may emerge endogenously.

Second, the disparity in profits between the leader and follower can be reduced (and a more "equitable" distribution of profits achieved) if the price leader keeps its price constant for some time. In this case, the leader faces a tradeoff. On the one hand, it would like to exploit the follower by responding strongly to current relative demand. On the other hand, if relative demand conditions are expected to revert to normalcy in a little while, such a strong response reduces future profits. Therefore, rigid prices reduce the response of the leader's price to current relative demand. Generally, however, the follower will not be in favor of completely rigid prices. Counterbalancing the profit-sharing benefit of inflexible prices is the advantage of letting the leader respond to common (and not merely relative) demand fluctuations. Somewhat (but not completely) rigid prices should therefore be expected to be a feature of a price leadership regime.

Third, social welfare (measured as the sum of consumer and producer surplus) is even lower under price leadership than under overt collusion! While the duopolists are able to achieve a somewhat collusive outcome using price leadership, they clearly do not do as well as they would if they were completely unconstrained, i.e., if they could sign a binding contract and could enforce honest revelation of private information. Consumers, on the other hand, are generally better off under a price leadership regime than they are when faced with overt collusion. When demand is relatively strong for the leader's product, the leader sets a high price, and thus consumer

surplus from each product is low compared with that under full collusion. The opposite is true when demand for the leader's product is relatively weak. However, the increase in consumer surplus when demand for the leader's product is relatively weak exceeds the reduction in consumer surplus when it is relatively strong. This is because consumer surplus is a convex function of price.

The collusive interpretation of price leadership suggests that it warrants antitrust scrutiny. The U.S. Courts, however, have taken a very accomodating attitude. As early as 1927 in U.S. v. International Harvester, the U.S. Supreme Court stated:

"[International Harvester] has not .. attempted to dominate or in fact controlled or dominated the harvesting machinery industry by the compulsory regulation of prices. The most that can be said as to this, is that many of its competitors have been accustomed, independently and as a matter of business expediency, to follow approximately the prices at which it has sold its harvesting machines....And the fact that competitors may see proper, in the exercise of their own judgement, to follow the prices of another manufacturer, does not establish any suppression of competition or show any sinister domination."<sup>3</sup>

Our theory has three empirically verifiable implications. First, an industry that moves from price competition to a price leadership regime should become more profitable. Second, the price leader should enjoy an even greater increase in profitability than its rivals by virtue of its ability to select the prices that will be charged. Finally, if the firms differ in the quality of their information about demand conditions, a relatively well-informed firm should emerge as the price leader.

An industry that attempts to make this change from competition to price leadership therefore provides a convenient natural experiment through which the validity of our theory can be assessed. One notable modern episode of this kind is the March 1983 attempt by American Airlines to lead the industry to a new fare structure. After the deregulation of the domestic US airline industry in the late 1970's and early 1980's,<sup>4</sup> the industry went through a period of upheaval in

which price warfare was common.<sup>5</sup> Then, without prior notice, on March 14, 1983 American Airlines announced a new fare plan to become effective starting April 2.<sup>6</sup> The plan replaced thousands of different fares with four basic mileage-based fares.<sup>7</sup> American expressed its desire for other airlines to match its fares and their response was quick and positive. United adopted the plan the following day and by March 17 most other airlines had adopted it too.<sup>8</sup>

We present evidence that suggests that American's computer reservation system (SABRE) provided it with detailed market demand information that was unsurpassed in the industry. We also use stock market data to examine the effect on the equity values of the airline companies of American's announcement and the industry's response. Our results indicate that the increase in those values was too unusual to be attributed to the random fluctuations of the stock market. Furthermore, the airline that realized the largest single gain in the value of its equity was none other than American Airlines, a finding that is consistent with the theory. Thus all three of our theory's implications are broadly borne out by this historical episode.

The paper treats the subjects in the following order: the necessary conditions for existence of a collusive price leadership equilibrium (Section II), price rigidity (Section III), welfare (Section IV), and the analysis of the American Airlines fare plan (Section V). Section VI provides concluding remarks.

## II. THE MODEL

We consider a duopoly in which firm 1 has better information about demand conditions than firm 2. Both goods are produced with constant marginal cost  $c$ . The demands for goods are assumed to be given by:

$$Q_1 = x - bP_1 + d(P_2 - P_1)$$

$$Q_2 = y - bP_2 + d(P_1 - P_2)$$

where  $Q_i$  and  $P_i$  are the quantities demanded and price of good  $i$  respectively. The parameters  $x$  and  $y$  vary over time while  $b$  and  $d$  are positive constants. The constant  $b$  gives the response of the quantity demanded of each good to a fall in the price of both goods while  $d$  gives the response to a relative increase in the other good's price. Note that  $d$  is zero if the demand for both goods is independent while it is infinite if the goods are perfect substitutes.

It is convenient to decompose the demand disturbances into common and idiosyncratic components. Letting  $a=(x+y)/2$  and  $e=(x-y)/2$ , the above equations become:

$$\begin{aligned} Q_1 &= a+e-bP_1+d(P_2-P_1) \\ Q_1 &= a-e-bP_2+d(P_1-P_2). \end{aligned} \tag{1}$$

Note that in this form  $a$  and  $e$  are uncorrelated as long as the variance of  $x$  and  $y$  are the same.<sup>9</sup> The only lack of symmetry is given by  $e$  which raises the demand for good 1 relative to that of good 2.

We want to capture a situation in which the two firms have differential information about demand and in which one firm has somewhat better information. For simplicity, we assume instead that firm 1 knows both  $a$  and  $e$  while firm 2 is totally uninformed. Our main results, however, carry over to situations where firm 2 is also somewhat informed.<sup>10</sup> In our model, firm 2 knows only the mean of  $a$  ( $a'$ ) and the mean of  $e$  (zero). Moreover, we assume that there is no credible way for firm 1 to communicate its information.<sup>11</sup>

We begin by deriving the conditions under which price leadership, in which firm 1 is the first to announce its price, is sustainable as an equilibrium in this model. Unless it is punished for doing so, firm 2 will almost always undercut the price charged by firm 1. We assume that if firm 2 charges a price different from firm 1's price that the firms revert to competitive behavior forever.<sup>12</sup>

Suppose that firm 1 is the leader and it can be sure that firm 2 will follow its price  $P$  for the current period. Then firm 1 chooses  $P$  to maximize:

$$R_1 = (P - c)(a + e - bP). \quad (2)$$

Thus:

$$P = c/2 + (a + e)/2b, \quad (3)$$

and firm 1's profits are given by:

$$R_1 = [a + e - bc]^2/4b. \quad (4)$$

Now consider firm 2. Should it follow firm 1? Firm 2 has two incentives to deviate. First, when  $e$  is zero, it can increase its current profits by undercutting firm 1. Second, when  $e$  is nonzero, it prefers both  $P_1$  and  $P_2$  to differ from the price given by (3). Thus, to ensure that our form of price leadership is an equilibrium we must ensure that the benefits from continued cooperation outweigh the benefits from charging a price different from  $P$ .

The computation of whether firm 2 should deviate has four parts. First, we calculate firm 2's expected profits in future periods if it does not deviate in the current period,  $R_2$ . Second, we calculate firm 2's expected profits in future periods if it does deviate in the current period,  $\pi_2$ . The per period value of the punishment from deviating is then  $R_2 - \pi_2$ . Third, we calculate the current period profit to firm 2 if it deviates,  $\pi_{D0}$ . Finally, we calculate the current period profit to firm 2 if it does not deviate,  $\pi_{C0}$ . For it to be an equilibrium for firm 2 to match firm 1's price, the following condition must hold:

$$\pi_{C0} + \delta R_2 / (1-\delta) \geq \pi_{D0} + \delta \pi_2 / (1-\delta) \quad (5)$$

where  $\delta$  is the discount factor.

We turn now to the calculation of  $\pi_{C0}$ ,  $R_2$ ,  $\pi_{D0}$ , and  $\pi_2$ .

### (i) $R_2$

Firm 2's expected future per period profits from matching firm

1's price are given by:

$$R_2 = E[(a+e)/2b - c/2][a - e - (a+e)/2 - bc/2].$$

Since these are profits from the next period on, the expectation is taken before firm 2 learns firm 1's chosen P for each period.

Simplifying this expression gives:

$$R_2 = E[a - bc]^2/4b - 3Ee^2/4b.$$

Note that  $R_2$  is lower than  $R_1$ .

### (ii) $\pi_2$

If firm 2 fails to match firm 1's price, it triggers competition from the following period on. We assume that competition takes the form of the simultaneous-move one-shot game each period. Since firm 2 has no state dependent information it always announces the same price  $P_2$ . Thus firm 1 maximizes:

$$\pi_1 = (P_1 - c)(a + e - bP_1 + d(P_2 - P_1))$$

which leads to a price  $P_1$  equal to  $(a+e+dP_2+bc+dc)/2(b+d)$ . Firm 2, on the other hand, maximizes:

$$\pi_2 = E(P_2 - c)(a - e - bP_2 + d(P_1 - P_2))$$

where E is the expectations operator and  $\pi_2$  is the expected profit of firm 2. Thus firm 2 charges:

$$P_2 = (a' + bc + dc + dEP_1)/2(b+d) = (a' + (b+d)c)/(2b+d). \quad (6)$$

This implies that firm 1 charges:

$$P_1 = (a' + (b+d)c)/(2b + d) + (a - a' + e)/2(b+d). \quad (7)$$

The equilibrium value of  $\pi_2$  is:

$$\begin{aligned} \pi_2 &= [(a' + (b+d)c)/(2b + d) - c][a' - b(a' + (b+d)c)/(2b+d)] \\ &= (b+d)[(a' - bc)/(2b + d)]^2. \end{aligned}$$

Note that in the first equality the first term in brackets represents the difference between price and marginal cost while the second represents the average output of firm 2. For both of these magnitudes to be positive  $a'$  must exceed  $bc$ , which, in turn, must be positive if coordinated price increases are to reduce industry sales.

If  $a$  and  $e$  are i.i.d., the cost to firm 2 of the punishment is that it loses  $R_2 - \pi_2$  in every period starting with the next one. The discounted present value of these losses equals:

$$D' = ([E(a-a')^2 - 3Ee^2]/4b + (a'-bc)^2d^2/[4(2b+d)^2b])\delta/(1-\delta).$$

As we will see below, the first term in brackets actually represents the advantage of letting firm 1 be the leader instead of firm 2. The second term gives the excess of collusive over competitive profits when  $e$  is zero and  $a$  equals  $a'$ .  $D'$  is the difference in profits between being a follower in our price leadership model and refusing to cooperate. It must be positive for price leadership to be in both firms' interests.

### (iii) $\pi_{D0}$

After observing  $P$ , firm 2 becomes somewhat informed about  $(a-a')$  and  $e$  since it has an indirect observation of  $(a-a'+e)$ . Indeed, it knows that  $[2bP-bc-a'] \equiv x$  is equal to  $(a-a'+e)$ . Calculating its profits in the current period (whether from deviating or matching) therefore involves solving a signal extraction problem. Let  $s_a$  be the variance of  $a$  while  $s_e$  is the variance of  $e$ . Then, firm 2's expectation of  $(a-a')$  is equal to  $x s_a / (s_a + s_e)$  while its expectation of  $e$  is  $x s_e / (s_a + s_e)$ . If it were to charge  $P_2$  after firm 1 irrevocably announced the price given by (3) its expected profits would be:

$$(P_2 - c) \{a' + [2bP - bc - a'] (s_a - s_e) / (s_a + s_e) - bP_2 + d(P - P_2)\}. \quad (8)$$

If it deviates, firm 2 maximizes (8) which gives a price  $P'_2$ :

$$\begin{aligned} P'_2 &= \{a' + [2bP - bc - a'] (s_a - s_e) / (s_a + s_e) + dP + (b+d)c\} / 2(b+d) \\ &= \{2s_e a' + [bs_a + (b+d)(s_a s_e)] P + [ds_a + (2b+d)s_e]c\} / 2(b+d)(s_a + s_e) \end{aligned}$$

and expected profits of  $\pi_{D0} = (b+d)(P'_2 - c)^2$ . Since firm 2 tends to profit by undercutting firm 1,  $P'_2$  tends to be below  $P$ . As long as the variance of  $e$  is low enough, this is true for all  $P$ .

### (iv) $\pi_{C0}$

On the other hand, by not deviating, firm 2 earns the

expectation of (8) evaluated at P. This is the expectation of  $(P-c)(a-e-bP)$  conditional on P, which equals:

$$(P-c)[a's_e + b(s_a - 3s_e)P - bc(s_a - s_e)]/(s_a + s_e). \quad (9)$$

Substituting for  $\pi_{CO}$ ,  $R_2$ ,  $\pi_{D0}$ , and  $\pi_2$  in (5) gives the key necessary condition for existence of a collusive equilibrium:<sup>13</sup>

Condition A:

$$(b+d)(P'2 - c)^2 - (P-c) \frac{[a's_e + b(s_a - 3s_e)P - bc(s_a - s_e)]}{s_a + s_e} - D' < 0.$$

We now compare firm 2's profits when firm 1 is the leader with its profits if it were the leader itself.<sup>14</sup> If firm 2's profits are higher when firm 1 is the leader then the firms are unanimous in their choice of leader and it is natural to expect firm 1 to emerge as the leader.

If firm 2 is the price leader, it sets a price equal to  $(a' + bc)/2b$  which leads to expected profits given by:

$$L_2 = [a' - bc]^2/4b. \quad (10)$$

Thus the difference between  $R_2$  and  $L_2$  is simply

$$[E(a-a')^2 - 3Ee^2]/4b. \quad (11)$$

Therefore, firm 2 prefers to be a follower if the variance of a exceeds three times the variance of e. A high variance of a makes firm 2 want to be a follower since movements in a are incorporated into firm 1's prices. On the other hand variations in e are also incorporated in firm 1's prices, to firm 2's detriment. So a high variance of e makes firm 2 prefer to be a leader.

Qualitatively similar conclusions emerge if both firms are somewhat (although asymmetrically) informed. In that case one can derive a similar condition for when the better informed firm is the unanimous choice as price leader.

We conclude that price leadership has some natural advantages as a way of organizing a duopoly with asymmetric information. Not

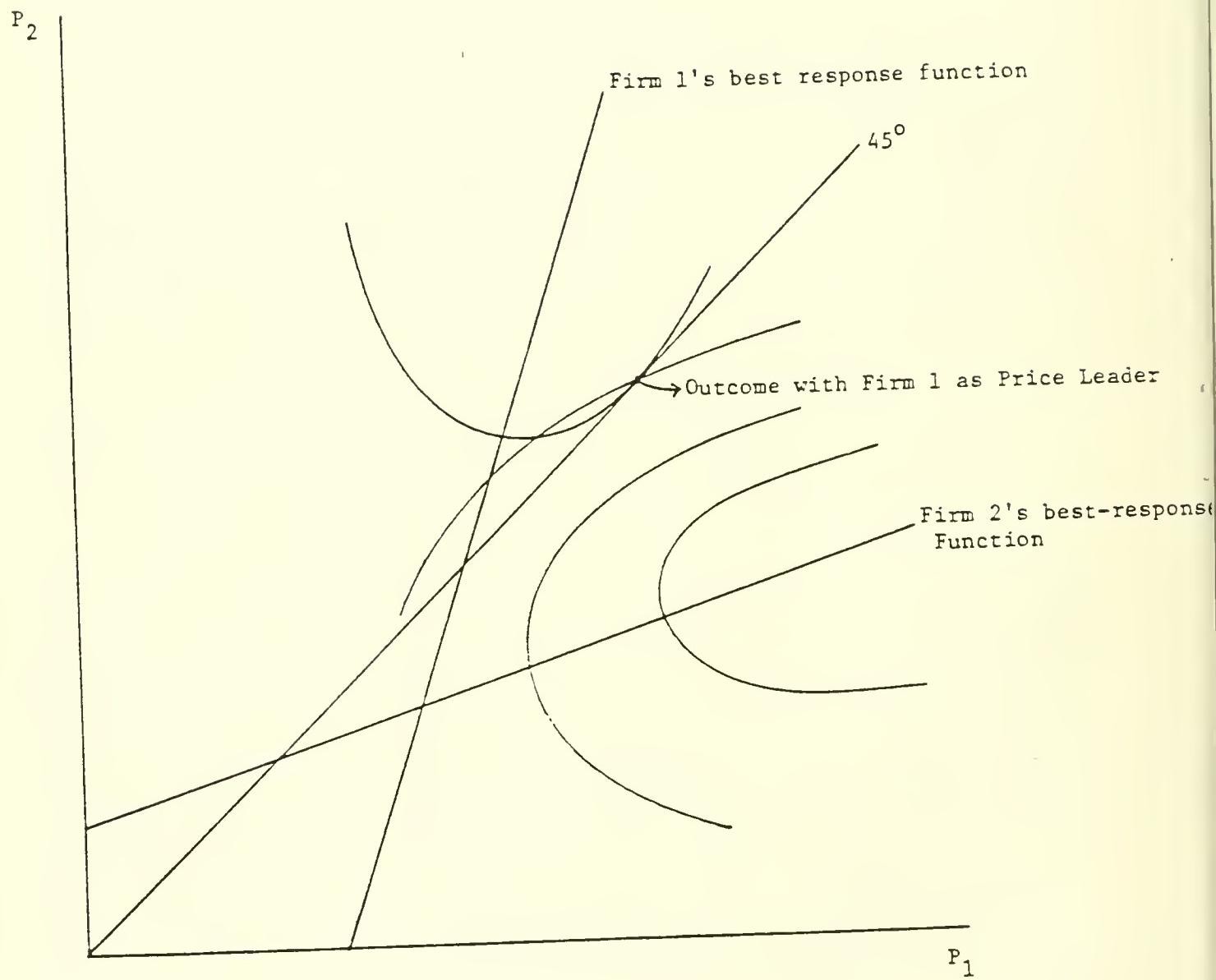
only does the leader have absolutely no incentive to deviate from his equilibrium strategy, but the detection of cheating by the follower is extremely simple. Moreover, in some circumstances the firms are unanimous in their choice of a leader.

Nonetheless it is important to note that the price leadership scheme does not replicate what the firms would be able to achieve under full information. Maximization of industry profits would dictate charging prices which maximize  $[Q_1(P_1-c)+Q_2(P_2-c)]$ . Using (1) these prices are equal to  $[a/b+c+e/(b+2d)]/2$  and  $[a/b+c-e/(b+2d)]/2$  respectively. These prices differ between firms as is to be expected. While maximization of industry profits should probably be regarded as too ambitious a goal for a duopoly which is unable to make side payments, price leadership also falls short of being Pareto for the two firms.

This lack of Pareto optimality can be seen with the aid of figure 1 which shows isoprofit lines for both firms in the space of  $P_1$  and  $P_2$ . These lines are drawn for  $e=e'>0$ . Therefore the tangency of an isoprofit line for firm 1 and the  $45^\circ$  line occurs at prices higher than the tangency of an isoprofit line of firm 2 and the  $45^\circ$  line. In our model of price leadership firm 1 picks the point at which one of its isoprofit lines is tangent to the  $45^\circ$  line. It is immediately apparent from the figure that both firms can be made better off if they both lower their prices with firm 2 lowering its price by more than firm 1 lowers its own. Furthermore, from the point of tangency of firm 1's isoprofit line with the  $45^\circ$  line a small reduction in both prices by the same amount raises total profits. This occurs because there is only a second order deleterious effect on firm 1's profits while the beneficial effect on firm 2 is of first order.<sup>15</sup>

In conclusion, price leadership achieves the optimal response to common changes in demand with great ease. Its cost (from the

FIGURE 1: The Suboptimality of Price Leadership Under Full Information



firms' point of view) is that prices do not respond optimally to changes in relative demand.

### III. PRICE STICKINESS AND PRICE LEADERSHIP

The problem, from firm 2's perspective, in allowing firm 1 to be the leader, is that firm 1 takes advantage of this and thus picks a price that rises proportionately to  $e$ , the difference in the two demand curves. This exploitation comes about not only when demands differ, but also when costs differ, as when firm 1 faces a strike by its workers. One way of mitigating this effect, particularly when the firms possess information of similar quality, is to let the firms alternate the leadership role.<sup>16</sup> An alternative way, and one that is more applicable when the firms' quality of information differs substantially and when temporary fluctuations in  $e$  are important, is to make prices relatively rigid. In other words, the leader is threatened with reversion to noncooperation if he changes his price too often. We study this role for rigid prices here.

If the leader must keep his price fixed for some time, it will make its price a function of current and expected future  $e$ 's. The longer the period of price rigidity, the more important are the expected future  $e$ 's when firm 1 sets its price. Accordingly, if the expectation of future  $e$ 's is relatively insensitive to current demand conditions, the presence of rigid prices dampens the effect of current  $e$  on price.

We illustrate this advantage of price rigidity with a simple example. In particular, we assume that  $e_t$ , i.e., the value of  $e$  at time  $t$ , is given by:

$$e_t = \beta e_{t-1} + \epsilon_t \quad (12)$$

where  $\beta$  is a number between zero and one while  $\epsilon_t$  is an i.i.d. random variable with zero mean. The intercept has two components; the first

of which is  $a'$ , a constant, while the second,  $a_t$ , moves over time according to the law of motion:

$$a_t = \phi a_{t-1} + \alpha_t \quad (13)$$

where  $\phi$  is a number between zero and one while  $\alpha$  is an i.i.d. random variable with zero mean. Thus the intercept,  $a' + a_t$ , tends to return to its normal value as well. To accomodate the existence of inflation we write the demand curve at  $t$  as:

$$\begin{aligned} Q_{1t} &= a' + a_t + e_t - b P_{1t} / S_t + d(P_{2t} - P_{1t}) / S_t \\ Q_{2t} &= a' + a_t - e_t - b P_{2t} / S_t + d(P_{1t} - P_{2t}) / S_t \end{aligned} \quad (14)$$

where  $S_t$  is the price level at  $t$ . This price level is given by:

$$S_t = \mu^t. \quad (\mu > 1) \quad (15)$$

The difference between  $\mu$  and one is the general rate of inflation. If the price leader must set a price that will be in force for  $n$  periods starting at time zero, it will pick a price that maximizes:

$$\begin{aligned} E_{10} \sum_{t=0}^n \delta^t &\left( \frac{P}{S_t} - c \right) (a_t + a' + e_t - b \frac{P}{S_t}) \\ &= \sum_{t=0}^n \delta^t \left( \frac{P_t}{\mu^t} - c \right) (\phi_t a_0 + a' + \beta^t e_0 - b \frac{P}{\mu^t}) \end{aligned} \quad (16)$$

where  $E_{10}$  is the expectation conditional on information available at time 0 to firm one and  $\delta$  is the real discount rate. This price,  $P(n)$  is given by:

$$P(n) = \frac{\frac{a_0}{2b} \frac{1 - (\delta\phi/\mu)^{n+1}}{1 - \delta\phi/\mu} + \frac{e_0}{2b} \frac{1 - (\delta\beta/\mu)^{n+1}}{1 - \delta\beta/\mu} + \frac{a'}{2b} + \frac{c}{2} \frac{1 - (\delta/\mu)^{n+1}}{1 - \delta/\mu}}{\frac{(1 - \delta/\mu^2)^{n+1}}{1 - \delta/\mu^2}}$$

Note that  $P(n)$  is increasing in  $n$  if there is inflation ( $\mu$  is greater than one). It is also increasing in  $\mu$  and  $e$ .

The expectation of the present discounted value of profits of the follower ( $W_2(n)$ ) is given by the expectation of the present value of profits for the  $n$  periods during which the leader keeps his price fixed ( $\pi(n)$ ) divided by  $(1-\delta^{n+1})$ .  $\pi(n)$  is given by:

$$\pi(n) = E_0 \sum_{t=0}^n \delta^t \left( \frac{P(n)}{\mu^t} - c \right) (a' + \phi t a_0 - \beta t e_0 - b \frac{P(n)}{\mu^t})$$

where  $E_0$  takes unconditional expectations. The unconditional expectation of both  $a_0$  and  $e_0$  is zero while their unconditional variance is  $\text{var}(\alpha)/1-\phi$  and  $\text{var}(\epsilon)/(1-\beta)$  respectively. This focus on unconditional expectations is warranted if the follower is completely uncertain about the state of demand at the moment he implicitly agrees to be a follower.

We can now write  $W_2(n)$  as:

$$W_2(n) = \frac{[a' + c^2 b]^2 [1-\delta/\mu^2] [1-(\delta/\mu)^{n+1}]^2}{4b[1-\delta/\mu]^2 [1-(\delta/\mu^2)^{n+1}] [1-\delta^{n+1}]} - \frac{a'c}{1-\delta}$$

$$+ \frac{\text{var}(\delta)[1-\delta/\mu^2][1-(\delta\phi/\mu)^{n+1}]^2}{4b(1-\phi)[1-\delta\phi/\mu]^2 [1-(\delta/\mu^2)^{n+1}] [1-\delta^{n+1}]}$$

$$- \frac{3\text{var}(\epsilon)[1-\delta/\mu^2][1-(\delta\beta/\mu)^{n+1}]^2}{4b(1-\beta)[1-\delta\beta/\mu]^2 [1-\beta/\mu^2]^{n+1} [1-\delta^{n+1}]}.$$

To evaluate the benefits of price rigidity we consider two special cases. In the first special case inflation is positive but the variance of  $\alpha$  is zero so that demand for the sum of the two products is deterministic<sup>17</sup> while in the second special case  $\mu$  is one (so that there is no inflation). In both cases there is an incentive to prolong the duration of prices, because this prolongation reduces the deleterious effect of  $\text{var}(\epsilon)$  on  $W_2$ . In both cases there is also a cost to long price durations. In the first case this cost is the

erosion in prices caused by inflation while in the second it is the insufficient response to changes in  $a$ . We analyze these special cases as follows. First, we give the conditions under which the follower would prefer some price rigidity, i.e., under which  $W_2(2) > W_2(1)$ . Then we study the numerical properties of  $W_2(n)$  for certain parameters.

Consider first the case in which inflation is zero. Assume also that  $\text{var}(\alpha)/[(1-\phi)(1-\delta\phi)^2]$ , which we denote  $\sigma_a$ , equals  $3\text{var}(\epsilon)/[(1-\beta)(1-\delta\beta)^2]$ , which we denote  $\sigma_e$ . Recall that this is the condition under which firm 2 is just indifferent between being a follower and a leader. Then, the follower prefers prices to be constant for two periods if:

$$\frac{[1-(\delta\phi)^2]^2}{1-\delta^2} - \frac{[1-(\delta\beta)^2]^2}{1-\delta^2} > \frac{[1-\delta\phi]^2}{1-\delta} - \frac{[1-\delta\beta]^2}{1-\delta} \quad (19)$$

As shown in the appendix this inequality is satisfied as long as  $\beta$  is less than  $\phi$ . Thus as long as the decay towards zero of the difference in demands is more rapid than the decay of the absolute level of demand towards its normal value, the follower prefers the leader to maintain some price rigidity. The intuition for this is that since a rapid decay of  $e$  towards zero means that the leader will be relatively inattentive to  $e$  when setting a price for a relatively long horizon. On the other hand, if  $a$  decays slowly, he will still make his price fairly responsive to the current value of  $a$ .

What we must show now is that the follower may prefer a finite period of price rigidity to an infinite one. This is plausible since, if  $e$  decays rapidly, the benefit from continued price rigidity, namely the loss in responsiveness to  $e$  becomes unimportant as the horizon becomes longer. We provide a numerical example in which the follower does indeed prefer a finite period of price rigidity.<sup>18</sup> Figure 2 shows

the value of  $W_2(n)$  when  $\sigma_a$  equals  $\sigma_e$  while  $k$  is .98,  $\beta$  is .6 and  $\phi$  is .9. The follower's welfare is maximized when  $n$  is equal to five. If, instead,  $\sigma_a$  is made to equal only  $.8\sigma_e$  then the maximum occurs at  $n = 7$ . Clearly, an increase in the variance of the difference in demands warrants a longer period of price rigidity to reduce further the effect of  $e$  on price.

Now consider the special case in which the  $\text{var}(\alpha)$  is zero. Assume further that,  $\sigma_e$  is equal to  $[(c^2 b + a')/(1-\delta/\mu)]^2/4b$  which we denote  $\sigma_\mu$ .<sup>19</sup> This case reveals a slightly different advantage of price rigidity. Price rigidity makes it impossible for the leader to fully respond in every period to increases in  $e$  even if these are permanent. Even when  $\beta$  is one, the follower benefits from this lack of responsiveness to  $e$ . This can be seen by noting that, under the current assumptions, when  $\beta$  is one,  $W_2$  is independent of both  $n$  and the rate of inflation. In this case the loss in the responsiveness to  $e$  just offsets the loss from responding to inflation more generally. If, instead,  $\beta$  is less than one, it can be shown that the follower always prefers some price rigidity. Indeed, it can be shown then that  $W_2(2) > W_2(1)$ . This requires that:

$$\frac{[1-(\delta/\mu)^2]^2}{(1-\delta^2/\mu^4)(1-\delta^2)} - \frac{[1-\delta\beta/\mu]^2}{(1-\delta^2/\mu^4)(1-\delta^2)} > \frac{(1-\delta/\mu)^2}{(1-\delta/\mu^2)(1-\delta)} - \frac{(1-\delta\beta/\mu)^2}{(1-\delta/\mu^2)(1-\delta)} \quad (20)$$

which is proved in the appendix. Thus, if the variance of  $\epsilon$  is sufficiently big while  $e$  decays even slightly towards its mean, firm 2 prefers some rigidity to complete flexibility. Once again, a decay of  $e$  over time induces the leader to make its price unresponsive to  $e$  if it is to keep a relatively rigid price. The follower benefits from this. We again consider some numerical examples to show that the follower may prefer a finite period of price rigidity. Figure 3 shows  $W_2(n)$  for  $\delta$  of .98,  $\beta$  of .7,  $\mu$  of 1.02 and  $\sigma_e$  equal to one fourth of

FIGURE 2: Profitability of Price Stickiness for the Follower -  
The Case of Stochastic Aggregate Demand

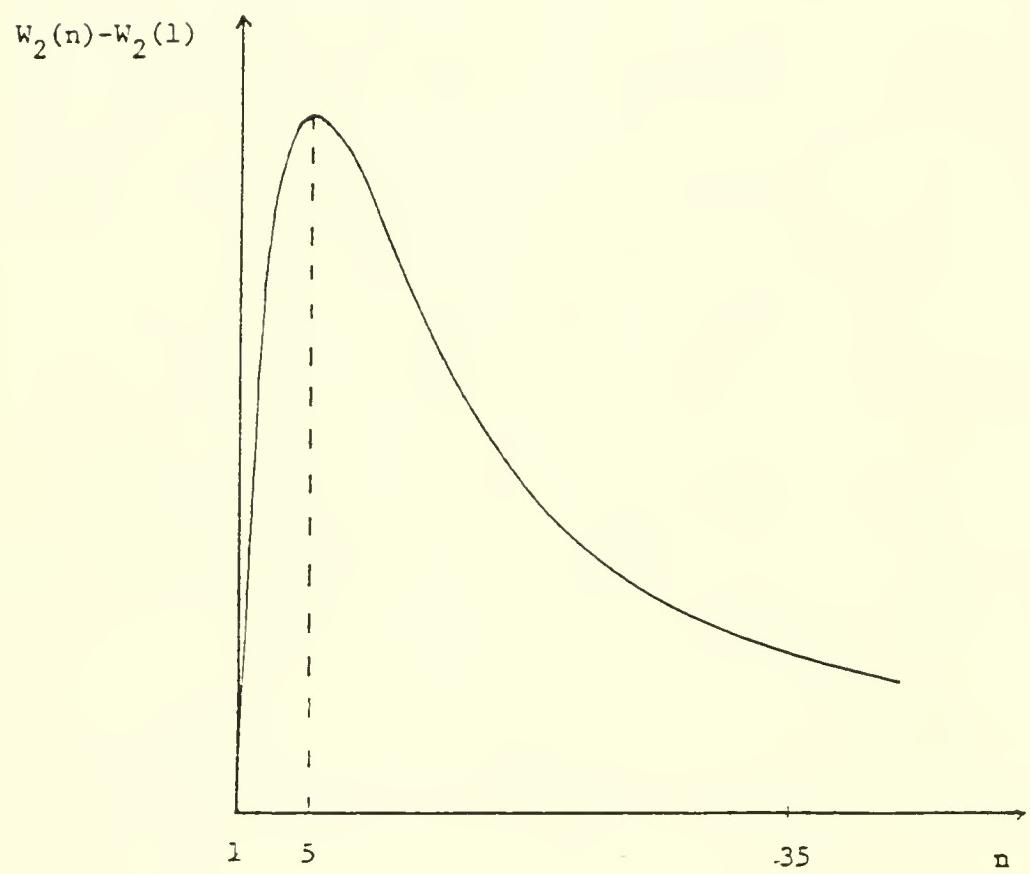
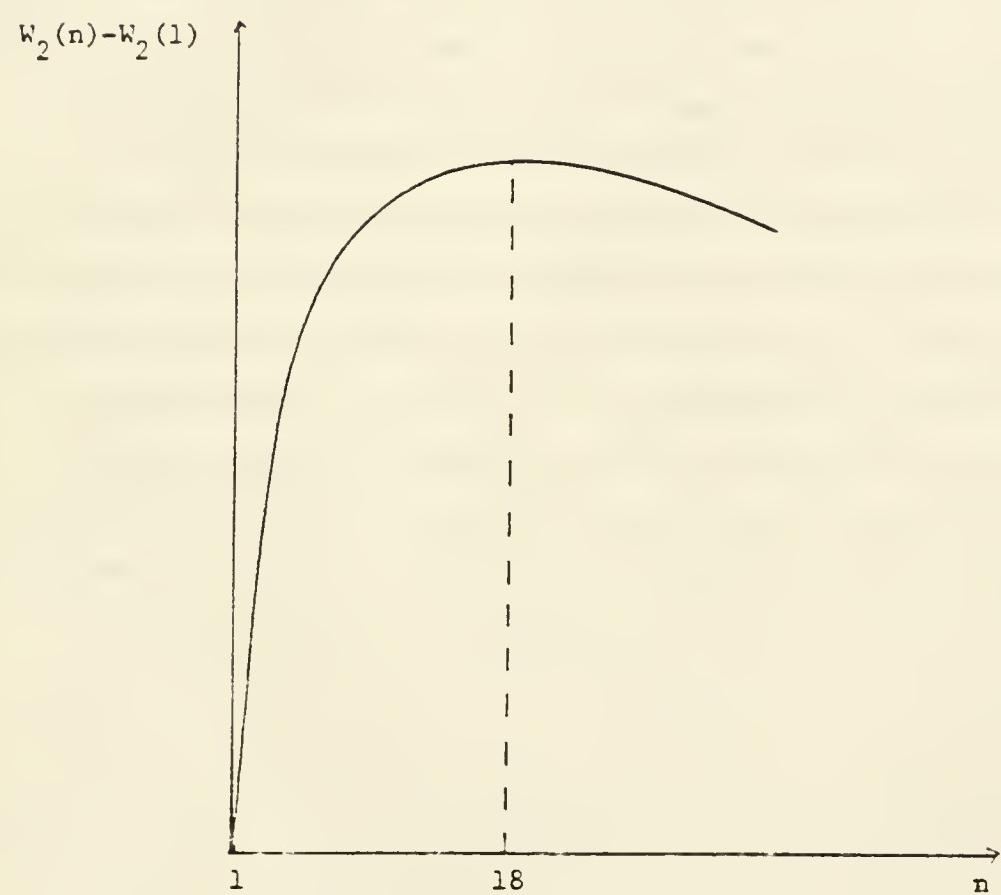


FIGURE 3: Profitability of Price Stickiness for the Follower - The Case of Inflation



$\sigma_\mu$ . The maximum is given by  $n=18$ . Reductions in inflation that make  $\mu$  equal to 1.01 raise the optimal  $n$ , from the follower's perspective, to 25. Increases in  $\sigma_e$  also tend to raise this optimal  $n$ .

While we have rationalized price rigidity by arguing that it leads to a more equitable distribution of the profits from implicit collusion one must be careful in drawing the implications of this analysis for macroeconomics. In particular, we have only shown that the follower wants to reduce the ability of the leader to respond to temporary shifts in relative demand (or cost). Thus, for instance, the follower does not want the leader to be able to take advantage of temporary concessions by the leader's suppliers. This argues for limits on the ability of the leader to change prices at will. However, it does not argue for complete nominal rigidity. Rather it argues for rules that make prices respond to aggregate shocks in a mechanical rather than a discretionary way at least for some time. It argues for some forms of indexation. It is only if such indexation is infeasible (because, for example, the antitrust authorities might become suspicious at such preannounced indexation or because customers prefer to be quoted a price rather than a price rule) that fixed nominal prices become the preferred solution to the oligopoly's problem.

#### IV. WELFARE CONSEQUENCES OF PRICE LEADERSHIP

Given that price leadership is an imperfect collusive device, one might think that it is not as bad from the point of view of overall welfare as overt collusion. In this section we compare welfare under price leadership and overt collusion. We saw in Section II that price leadership gives lower profits than overt collusion. Here we study mainly its effects on consumer welfare. We show that, by comparison with overt collusion, price leadership is better for

consumers but that overall welfare (as measured by the sum of producer and consumer surplus) is lower under price leadership.

In principle the analysis of consumer welfare should be carried out using the preferences of consumers whose aggregate demands are given by (1). In practice this is very difficult, in part because different individuals will be affected differently thus mandating interpersonal comparisons of utility, and in part because modelling the individual consumers whose demands aggregate to (1) is nontrivial. Therefore we compare instead the expected consumer surplus under the two regimes.

To simplify further we neglect variations in  $a$  and assume that  $c$  is zero. Also we assume that the distribution of  $e$  is symmetric. For this case the sum of producer and consumer surplus is the total area under the appropriate inverse demand curves. From (1) the inverse demand curves are given by:

$$P_1 = [a/b + e/(b+2d)] - Q_1(b+d)/[2(b+2d)] - Q_2d/[2b(b+2d)]$$

$$P_2 = [a/b - e/(b+2d)] - Q_2(b+d)/[2(b+2d)] - Q_1d/[2b(b+2d)].$$

To obtain the change in welfare from one equilibrium to another, i.e. from one pair of quantities to another, one integrates:

$$\int P_1(Q_1, Q_2) dQ_1 + P_2(Q_1, Q_2) dQ_2.$$

on a line from the first pair of quantities to the other. For our particular demand curves (and in general when compensated demand curves are used) the actual path of integration is irrelevant.<sup>20</sup>

When firms collude overtly,  $Q_1$  and  $Q_2$  maximize  $Q_1P_1 + Q_2P_2$ . Using (1) it is apparent that the outputs that maximize aggregate profits are given by  $(a+e)/2$  and  $(a-e)/2$  for firms 1 and 2 respectively. Under price leadership the corresponding quantities are instead  $(a+e)/2$  and  $(a-3e)/2$ . Thus the output of the leader is actually equal to his output under overt collusion and only the output of the follower is different. The effect of price leadership is to

make the follower's output very responsive to his demand. Thus it is very high when it is high (and  $e$  is very negative) and very low when demand is low ( $e$  high).

To compute the change in welfare resulting from the replacement of overt collusion with price leadership it is thus enough to integrate under the inverse demand curve for good two from  $(a-e)/2$  to  $(a-3e)/2$  holding  $Q_1$  at  $(a+e)/2$ . This gives:

$$-ae/2b - de^2/[2b(b+2d)] \quad (21)$$

For a positive  $e$ , follower output is lower under price leadership so welfare is lower. Of particular interest is the second term. As  $e$  becomes larger output of good 2 continues falling under leadership and the marginal units lost become more and more valuable. So price leadership becomes more than proportionately deleterious. The first term, on the other hand, is linear in  $e$ ; it leads to losses from price leadership when  $e$  is negative. Thus, in the case of a symmetric distribution of  $e$ , the first term does not affect the difference in average welfare between collusion and leadership. On the other hand the second term makes collusion more attractive for all  $e$ . Thus price leadership is worse on average.

The social losses from price leadership can be interpreted in yet another way. On average, follower output is equal to  $a/2$  both under leadership and under collusion. The main effect of leadership is to amplify the fluctuations in follower output. Since welfare tends to be a convex function of output, it declines on average as a result of these fluctuations.

While overall welfare is lower, we now show that consumers are better off. To do this it suffices to show that the fall in profits from moving to price leadership exceeds the fall in overall welfare. Prices under leadership are equal to  $(a+e)/2b$  so that total industry profits are

$$(a+e)[(a+e) + (a-3e)]/4b = a^2/2b - e^2/2b \quad (22)$$

Under overt collusion prices are given by  $[a/b+c+e/(b+2d)]/2$  and  $[a/b+c-e/(b+2d)]/2$  for firm 1 and firm 2 respectively. Thus overall profits are:

$$a^2/2b + e^2/2(b+2d) \quad (23)$$

Subtracting (22) from (23), cartel profits are bigger by:

$$e^2[1/b+1/(b+2d)]/2$$

which is larger than the loss in overall welfare (21) once one ignores the term linear in  $e$ . From the point of view of consumers, price leadership has the disadvantage that, when  $e$  is high, both prices are high so there is little surplus in either market. On the other hand when  $e$  is negative, both prices are low so surplus is high, particularly in the market for the preferred good (good 2). This latter effect dominates because the reduction in the price of good 2 when good 2 is the preferred good (which is given by  $e(b+d)/[b(b+2d)]$ ) is larger than the increase in the price of good 1 when good 1 is the preferred good (which is given by  $ed/[b(b+2d)]$ ).

## V. PRICE LEADERSHIP IN THE U.S. AIRLINE INDUSTRY

Our theory has three implications for the attempt by American Airlines to establish price leadership in the U.S. airline market: we would expect American Airlines to be relatively well-informed; for the announcement and adoption of the plan to have been profitable for all the airlines; and for American Airlines to have had a disproportionately large increase in profits. We provide evidence in support of each of these implications in turn below.

First, in terms of information, the larger carriers are presumably also the better informed about demand conditions since they observe passenger bookings on more routes and at different times. It is thus reassuring for the theory that the airline to emerge as price

leader is one of the two largest, second only to United Airlines. More importantly, however, is the access of some of the airlines to information generated by their computer reservation systems (CRSs). In 1983 there were six CRSs, including American's SABRE system and United's Apollo system. Bookings made on CRSs accounted for 88% of the domestic revenues of travel agencies.<sup>21</sup> Significantly, from our point of view, the SABRE system had a 49% market share of the domestic revenues generated by travel agencies using these systems. United's Apollo system, by contrast, had a market share of only 31%.<sup>22</sup> These systems were able to produce reports of bookings by carrier and by segment.

Furthermore as the Department of Justice reports "the marketing people had access to that information" and "it was used for planning purposes ..".<sup>23</sup> Moreover, at the time, American and United shared only a fraction of this information with other airlines, and then only with some delay.<sup>24</sup> Therefore American may have had the best demand information at the time that the new fare structure was proposed; at any rate, it seems implausible that any airline had significantly superior information.

Second, we study whether the announcement and acceptance of the American plan was profitable for the airlines. We do this by computing the change in the market value of the airlines' equity. Under assumptions standard in the field of Finance, this change in market value represents the change in the expected present discounted value of the stream of profits.<sup>25,26</sup> We ask whether the returns to the equities of the airlines were unusually high following the announcement.<sup>27</sup> This can be tested if one has an estimate of the variability of the risk-adjusted returns on the airlines' equity.

To obtain such an estimate, we start by fitting the simple capital asset pricing model (CAPM) to the 200 trading days before

March 14 and to 200 trading days starting with the day 56 trading days after March 14. This gives us estimates of the  $\alpha$ 's and the  $\beta$ 's of each of the airlines as well as of the portfolio which consists of an equal number of dollars invested in each airline.<sup>28</sup> Using these estimates we can compute the excess returns (the returns not explained by the  $\alpha$ 's or by the movements in the stock market as a whole.<sup>29</sup>) during the days following March 14. An estimate of the variance of these excess returns is computed using the 50 trading days starting on March 22, by which time the effect of the airlines' announcements was presumably incorporated in the stock prices.<sup>30</sup> The first two columns of Table 1 present the excess returns for the periods March 14-17 and March 15-17.<sup>31</sup>

Since we are interested in knowing whether these returns are unusual for the industry as a whole, we first abstract from the idiosyncratic risk of each airline. To do this we study the excess returns on an equally weighted portfolio of all fourteen airlines. This return is significantly positive at the 1% level when considering the three day return and at the 5% level when considering the four day return. Moreover, and perhaps more strikingly, the excess returns of each individual airline were positive in this period, some significantly so.

The newspapers of the period do not suggest any alternative reason for the abnormal rises in airlines stock prices. Oil prices were relatively quiescent and actually rose slightly (which would be expected to reduce airline stock prices) in the period under analysis. The contract for April 83 delivery of leaded gasoline had a delivered price of 76.6 c/gallon at the open of 3/14 and a price of 79 c/gallon at the close of 3/17.

TABLE 1  
EXCESS RETURNS AND INCREASED MARKET VALUE

<u>Airline</u>	3/15-3/17 return	3/14-3/17 return	3/15-3/17 increase in value*
American Airlines	0.12 (2.65)	0.11 (2.10)	109,212
Delta Air Lines	0.04 (1.29)	0.04 (0.91)	83,050
Eastern Air Lines	0.04 (0.63)	0.04 (0.61)	8,240
Northwest Air Lines	0.06 (2.00)	0.05 (1.54)	60,533
Ozark Holdings	0.05 (1.78)	0.03 (1.00)	8,955
PSA	0.06 (1.59)	0.06 (1.38)	6,941
Pan Am	0.05 (0.97)	0.08 (1.28)	4,884
Piedmont Aviation	0.02 (0.77)	0.01 (0.53)	6,705
Republic Airlines	0.16 (3.82)	0.13 (2.80)	30,820
Texas Air	0.04 (0.66)	0.02 (0.26)	4,593
Trans World Airways	0.09 (2.78)	0.10 (2.63)	44,617
United Air Lines	0.09 (2.78)	0.08 (2.23)	91,576
US Air	0.04 (1.02)	0.05 (1.16)	29,589
Western Air Lines	0.04 (0.72)	0.01 (0.11)	3,748
Equally weighted portfolio	0.06 (3.14)	0.05 (2.07)	
Total			493,463

t-statistics in parenthesis

\*In thousands of dollars

Finally, we examine the absolute change in the value of the airlines' equity. The last column in the table gives the change in market value implied by the excess returns. As can be seen, American's increase in value is the largest (\$109.2m), exceeding even that of the largest carrier, United (\$91.5m). Also, the total change in market value (which understates the gains to all airlines both because untraded carriers are excluded and because preferred shares are ignored) is almost \$0.5b suggesting that price leadership is a quite effective collusive device.

These facts appear to be consistent with all three of the implications of our theory. Yet it is important to be cautious in interpreting this historical episode as a transition from disorganization to price leadership. In some sense this was just a fare change, like many others. Indeed many regular coach fares actually fell. However, a major feature of the American proposal was to eliminate discounting from coach fares, except in narrow special circumstances. American's stated goal was to lower the number of passengers traveling on discount fares from 85% to 50%.<sup>32</sup> This feature of increased discipline in the industry, supported by the threat to match lower prices,<sup>33</sup> is one of the hallmarks of the industrial statesmanship that characterizes price leadership. Similarly, it is American's insistence that the change in fare structure be an industry-wide one, that strongly suggests it viewed itself as a price leader.

#### IX. CONCLUDING REMARKS

This paper has explored the possibility that price leadership is a collusive device in industries in which firms have private information. In such a setting price announcements go part way

towards providing the kind of information revelation the firms could achieve if they could meet to share information and fix prices. The key features of the model are these: the firms may be unanimous in their choice of price leader even though the price leader is in an advantageous position; price rigidity may serve as a device for decreasing the dispersion in the profits of the firms; and, from a welfare point of view, price leadership is worse than overt collusion.

While the price leadership scheme has many advantages from the point of view of the duopoly, it does not usually implement outcomes on the duopoly's profit frontier. For example, rather than insisting on exact price matching, the firms could adopt a scheme in which the second firm picks a price which is a function of the announced price of the first firm.

To prevent the second firm from simply undercutting the first firm, the precise function must be understood by both firms, and the second firm must be punished (by a period of noncooperation) if it deviates from this function. Typically, where the products are differentiated, an optimal function of this type would lead to different prices across firms. In practice, the function that gives the second firm's price as a function of the first would probably have to be quite complicated and would ideally have to change over time as conditions change. For punishments not to be triggered by accident, both firms must understand the exact form of this function.

Two other forms of price leadership are discussed in the Industrial Organization literature. The first, "barometric" price leadership, refers to a situation in which the price leader merely announces the price that would prevail anyway under competition. In contrast to the collusive price leader, the barometric price leader has no power to (substantially) affect the price that is charged generally in the industry. Indeed the actual price being charged may

soon diverge from that announced by the barometric firm, which in turn is unable to exert any disciplining influence to prevent this from occurring. In markets with differentiated products, therefore, the barometric model does not seem very persuasive.

The other form of price leadership, and the one which has been the focus of most formal modelling, is the one that results from the existence of a dominant firm. Models of this type (see Gaskins (1971) and Judd and Petersen (1985)) assume that the dominant firm sets the price of a homogeneous product. This price is then taken as given by a competitive fringe of firms. Unfortunately, this model cannot explain the behavior of oligopolies in which there are several large firms. Such large firms cannot be assumed to take as given the price set by any one firm. Rather, they should be expected to act strategically.<sup>34</sup>

A collusive interpretation of price leadership casts some doubt on the reasonableness of the current legal status of price leadership. However, anyone who would render price leadership a criminal act must be sensitive to the possibility that there is a benign explanation. In particular, if one is to condemn a firm simply because its rivals choose to charge the same price as it does, one must be fairly convinced that the price matching is a collusive practice. This suggests that additional work on noncollusive theories of price matching is needed.

FOOTNOTES

- 1 See Nicholls (1951) for a detailed discussion of pricing in the cigarette industry during this period.
- 2 See Stigler (1947) for a discussion and further references.
- 3 U.S. v. International Harvester Co., 274 U.S. 693. 708-709 (1927). See also Scherer (1980) p. 520 ff.
- 4 The Airline Deregulation Act which was signed on October 28, 1978 proposed a gradual relaxation of the Civil Aeronautics Board's regulation over a four year period.
- 5 For an analysis of this period see Bailey, Graham and Kaplan (1985).
- 6 "American to Base Fares on Mileage," New York Times, March 15. 1983.
- 7 These fares were first class, coach, a 25% discount for off-peak travel and a Super Saver fare. The Super Saver fare required advance purchase and had a minimum required length of stay.
- 8 "Most Big Airlines Back American's Fare Plan", New York Times, March 17, 1983.
- 9 Changes in  $\epsilon$  are akin in their effect to changes in the relative marginal costs of the two firms.
- 10 See Rotemberg and Saloner (1985) for a formal treatment of the case where both have some information.
- 11 Note that firm 1 has an incentive to lie about its information. If it could convince firm 2 that its demand is large, it could induce it to charge a high price thus increasing demand for its own product.
- 12 As Abreu (1982) has shown such punishments are not generally optimal. Larger punishments can be achieved by punishing deviations from the punishment strategy. Making this modification would have no material affect on our results.

13 This condition assumes that firm 1 punishes firm 2 both for downwards and upwards deviations in its price. Punishment for upwards deviations seems unreasonable. As mentioned above however, under plausible conditions firm 2 always undercuts firm 1 when it deviates.

14 In this particular model, firm 2 is uninformed, so this is equivalent to having the duopoly agree on state independent prices.

15 On the other hand the tangency of an isoprofit line of firm 2 with the  $45^\circ$  line can be interpreted as the price picked by firm 1 as the leader when  $\epsilon$  is equal to  $-e'$ . There, the prices are too low. Both firms would benefit from price increases with the follower increasing its price more than the leader.

16 Alteration of this kind has been observed in a variety of industries, including steel and cigarettes.

17 Of course, in a model in which the variance of  $\alpha$  is literally zero, our rationale for price leadership disappears. However, this example is only intended to illustrate the effects of inflation on price rigidity.

18 This preference for finite periods of price rigidity is a feature of every numerical example we have studied.

19 This is probably unrealistic since it requires a very large variance of  $\epsilon$ .

20 This occurs because our inverse demand functions are the partial derivatives of a function of  $Q_1$  and  $Q_2$ . In our case that function is quadratic. See Diamond and McFadden (1974) for a general discussion.

21 Advance Notice of Proposed Rulemaking - Airline Computer Reservation Systems", Comments and Proposed Rules of the Department of Justice before the CAB, Docket 41686, September 9, 1983.

22 "1985 Report of the Department of Justice to Congress on the Airline Computer Reservation System", U.S. Department of Justice, December 20, 1985, Appendix.

23 U.S. Department of Justice, 1983, *ibid.*, p .155.

24 *ibid.* pp. 156/7.

25 See Fama, Fisher, Jensen and Roll (1969) and Ruback (1982) for examples of our methodology. Rose (1985) applies this method to the analysis of other aspects of industry structure.

26 To the extent that investors were uncertain that the American plan would succeed, the change in market value under estimates the value of successful price leadership. The change in market value thus provides a lower bound on this value.

27 The data for this analysis are collected by the Center for Research in Security Prices of the University of Chicago.

28 Our results do not depend on whether we use the estimates obtained from only the trading days after the event or those obtained using both subsamples. We report results based on the latter procedure.

29 The market as a whole refers to the NYSE and the AMEX.

30 Our estimate of the variance is based on the assumption that the true variance is constant. We also allowed the variances to change in an autocorrelated manner as in Scholes and Williams (1977) without materially affecting the results.

31 The returns are based on closing quotations. Thus a 0.12 return from March 15 to March 17 means that \$1.00 invested at the close of March 14 yielded \$1.12 at the close of March 17. We focus on returns starting both on March 14 and on March 15 because we do not know whether the American Airlines announcement took place before or after the stock markets closed.

32 "Prospects for New Air Fares", New York Times, March 30, 1983.

33 New York Times, March 15, *ibid.*

34 The identical criticism can be applied to the model of d'Aspremont et al (1983). There, a group of equal-sized firms collude

to set the price; the remaining firms, which are assumed to be of the same size, treat this price parametrically. Since this explicitly assumes that the fringe firms are large, the above criticism is especially relevant.

APPENDIX I: PROOF OF INEQUALITIES 19 AND 20

Since,  $\delta$ ,  $\beta$  and  $\phi$  are less than one inequality (19) can be written as follows:

$$\frac{(\delta\beta)^2 - (\delta\phi)^2}{1-\delta^2} > \frac{\delta\beta - \delta\phi}{1-\delta} X$$

where

$$X = \frac{(2 - \delta\phi - \delta\beta)(1 + \delta)}{2 - (\delta\phi)^2 - (\delta\beta)^2}$$

This is equivalent to:

$$\frac{\delta(\beta-\phi)}{1-\delta} \left[ \frac{\delta\beta + \delta\phi}{1+\delta} - X \right] > 0.$$

So, if  $\beta < \phi$  the inequality is satisfied as long as  $X$  is bigger than  $(\delta\beta + \delta\phi)/(1+\delta)$  which is obviously less than one. Yet  $X$  is bigger than one since by subtracting the denominator of  $X$  from its numerator one obtains:

$$2 - \delta\phi - \delta\beta + 2\delta - \delta^2\phi - \delta^2\beta - [2 - (\delta\phi)^2 - (\delta\beta)^2] = \\ \delta(1-\phi)(1-\delta\phi) + \delta(1-\beta)(1-\delta\beta) > 0$$

This completes the proof.

To prove the inequality given by (20) we first note that, since  $\delta$  is smaller than one while  $1/\mu$  and  $\beta$  are at most equal to one, (20) can be written as:

$$\frac{(\delta\beta/\mu)^2 - (\delta/\mu)^2}{1 + \delta/\mu^2} > (\delta\beta/\mu - \delta/\mu) X'$$

where:

$$X' = \frac{(2 - \delta\beta/\mu - \delta/\mu)(1+\delta)}{2 - (\delta\beta/\mu)^2 - (\delta/\mu)^2}$$

This is equivalent to:

$$(\delta\beta/\mu - \delta/\mu) \left[ \frac{\delta/\mu + \delta\beta/\mu}{1 + \delta/\mu^2} - X' \right] > 0$$

So, if  $\beta$  is smaller than one the inequality is satisfied as long as  $X'$  is bigger than  $(\delta/\mu + \delta\beta/\mu)/(1 + \delta/\mu^2)$ . This latter expression is smaller than one since by subtracting the denominator from the numerator one obtains:

$$\begin{aligned} & (\delta/\mu)(\beta-1) + 2\delta/\mu - [1-\delta + \delta + \delta/\mu^2] \\ &= (\delta/\mu)(\beta-1) - (1-\delta) - \delta(1-1/\mu)^2. \end{aligned}$$

Moreover  $X'$  is greater than one since, by subtracting its denominator from its numerator one obtains:

$$\begin{aligned} & (2-\delta\beta/\mu-\delta/\mu)(1+\delta) - (2-(\delta\beta/\mu)^2 - (\delta/\mu)^2) = \\ & (1-\beta/\mu)(\delta-\delta^2\beta/\mu) + (1-1/\mu)(\delta-\delta^2/\mu) > 0 \end{aligned}$$

This completes the proof.

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